Stress transfer along crack

-Contact density model-

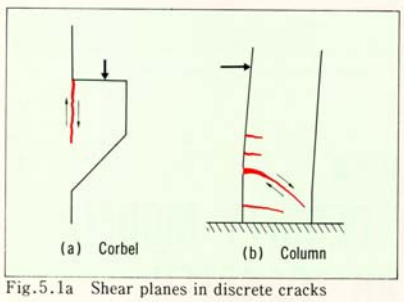


Fig.5.1a Shear planes in discrete cracks

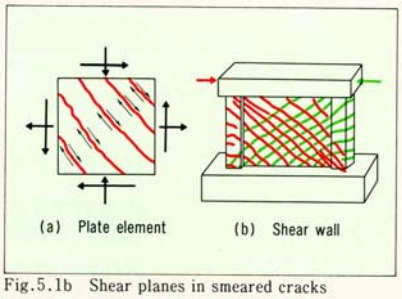
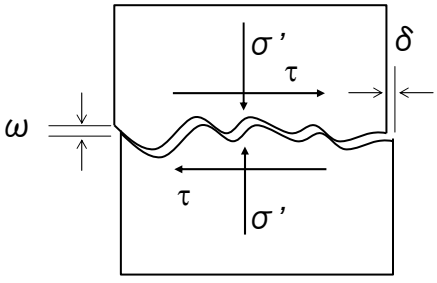


Fig.5.1b Shear planes in smeared cracks

Stresses and displacements along crack

Concrete



σ' : normal stress

τ : shear stress

δ : shear displacement

ω : crack opening

Experiment

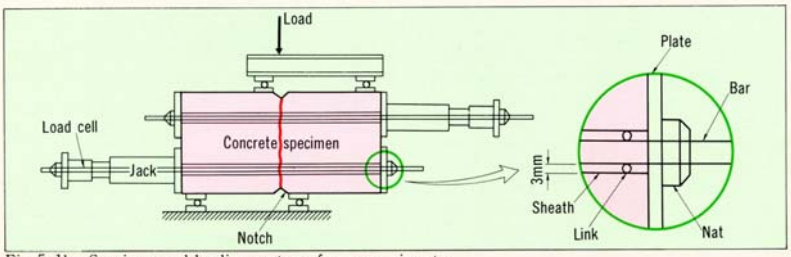


Fig.5.1k Specimen and loading system of our experiments

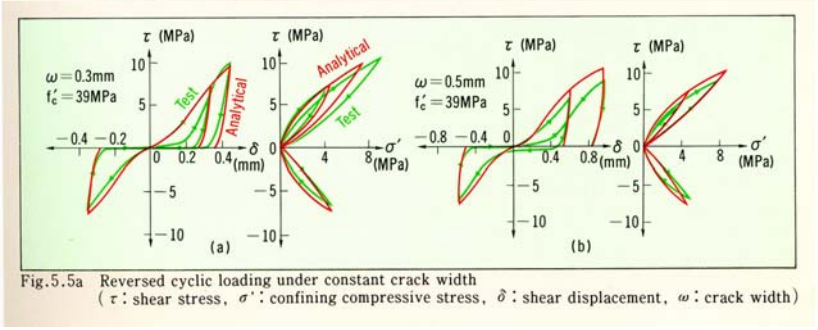
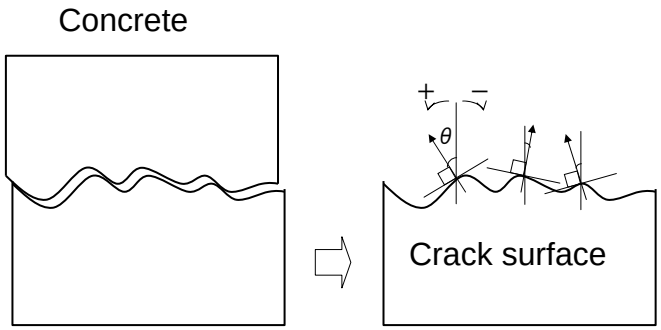


Fig.5.5a Reversed cyclic loading under constant crack width
(τ : shear stress, σ' : confining compressive stress, δ : shear displacement, ω : crack width)

Geometry of crack surface



θ : crack surface direction

Measurement and modeling of crack geometry

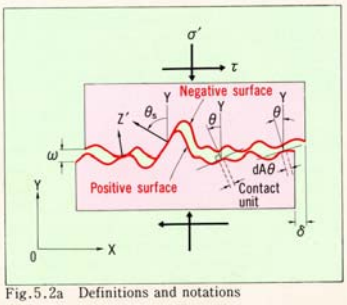


Fig.5.2a Definitions and notations

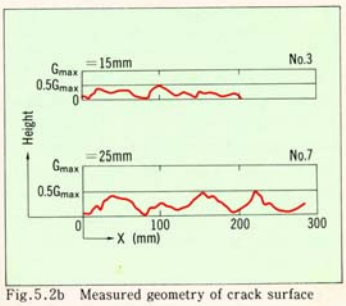


Fig.5.2b Measured geometry of crack surface

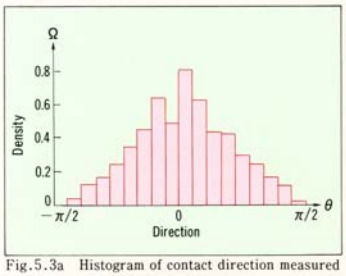


Fig.5.3a Histogram of contact direction measured

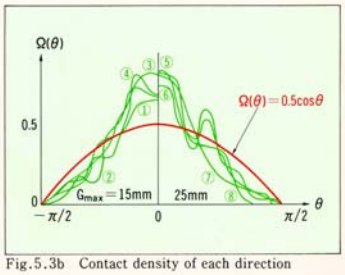


Fig.5.3b Contact density of each direction

Contact density function

$$dA_\theta = A_t \Omega(\theta) d\theta$$

dA_θ: Area of contact unit having inclination angle between θ and $\theta+\Delta\theta$

$$\int_{-\pi/2}^{\pi/2} dA_\theta \cos\theta = 1$$

A_t: Whole surface area per unit crack plane

$$= \left\{ \int_{-\pi/2}^{\pi/2} \Omega(\theta) \cos\theta \cdot d\theta \right\}^{-1}$$

$\Omega(\theta)$: Contact density function representing distribution of crack surface direction

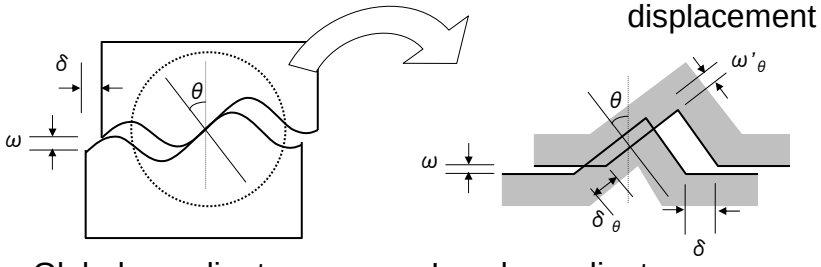
$$\int_{-\pi/2}^{\pi/2} \Omega(\theta) d\theta = 1$$

Local displacement

$$\omega'_\theta = \delta \sin\theta - \omega \cos\theta$$

$$\delta_\theta = \delta \cos\theta + \omega \sin\theta$$

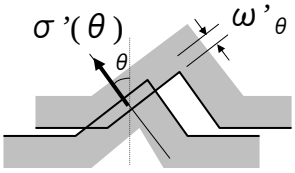
ω'_θ : Contact displacement

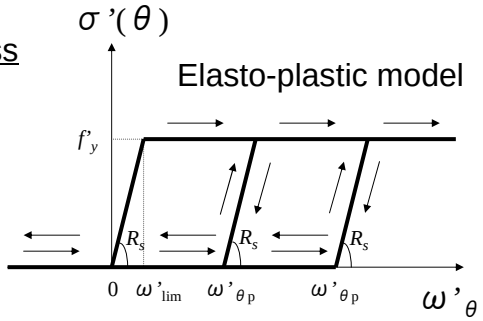


Global coordinate (ω, δ)

Local coordinate (ω'_θ, δ_θ)

Contact displacement – Contact compressive stress





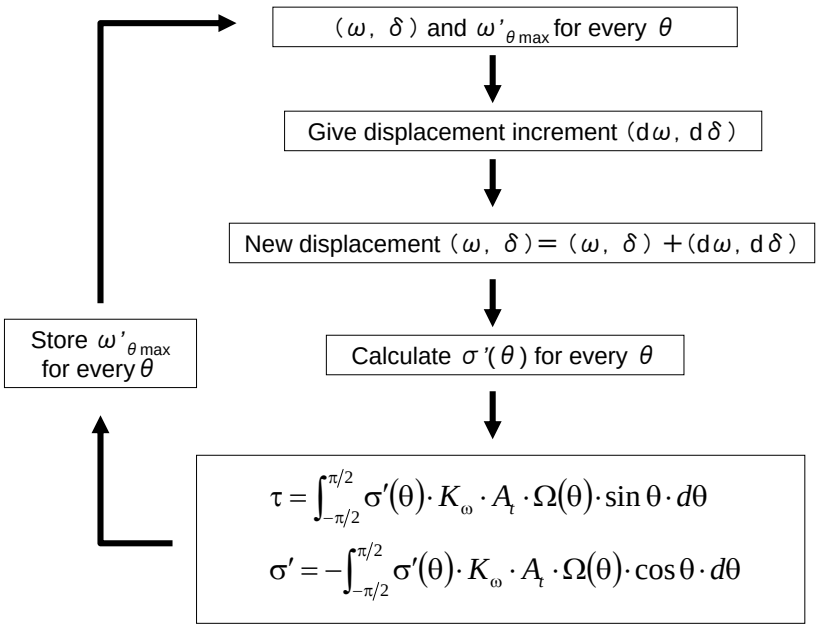
$$\sigma'(\theta) = \begin{cases} R_s\left(\omega'_\theta - \omega'_{\theta p}\right) & \left(in \quad \omega'_\theta \geq \omega'_{\theta p}\right) \\ 0 & \left(in \quad \omega'_\theta < \omega'_{\theta p}\right) \end{cases}$$

Where,

$$\omega'_{\theta p} = \begin{cases} \omega'_{\theta \max} - \omega'_{\lim} & \left(in \quad \omega'_{\theta \max} \geq \omega'_{\lim}\right) \\ 0 & \left(in \quad \omega'_{\theta \max} < \omega'_{\lim}\right) \end{cases}$$

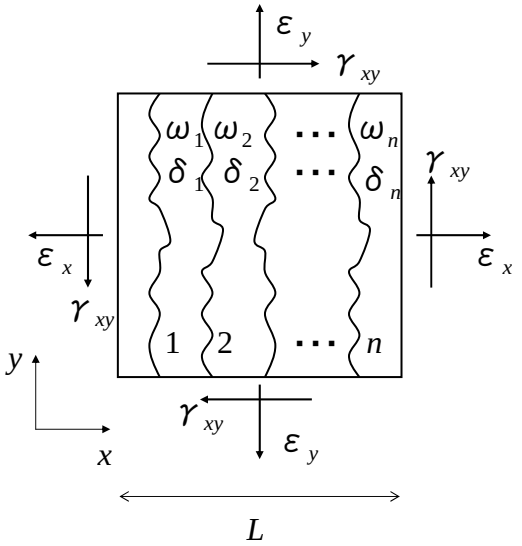
$\omega'_{\theta\,max}$: maximum of ω'_θ ever experienced

ω'_{lim}, R_s, f'_y : material constants



Smeared crack modeling

Average strain of cracked concrete



$$\begin{cases} \varepsilon_x \cong \dfrac{\sum_{i=1}^{n} \omega_i}{L} = \dfrac{n}{L}\overline{\omega} \\ \gamma_{xy} \cong \dfrac{\sum_{i=1}^{n} \delta_i}{L} = \dfrac{n}{L}\overline{\delta} \end{cases}$$

$$\Leftrightarrow$$

$$\begin{cases} \overline{\omega} = \dfrac{L}{n}\varepsilon_x \\ \overline{\delta} = \dfrac{L}{n}\gamma_{xy} \end{cases}$$

L/n: average crack spacing

Assignment

Calculate τ and σ' under a given deformation path. Draw τ - δ diagram and τ - σ' diagram. Discuss characteristics of stress transfer behavior of concrete crack on the basis of the obtained computational results.

(1) ω : 0.5mm (constant)
δ : 0 → 0.3mm → (unloading) → (τ =0) → (reloading) → 0.5mm → (unloading) → -0.3 mm → (unloading) → (τ =0)

(2) ω : 1.0mm (constant)
δ : 0 → 0.3mm → (unloading) → (τ =0) → (reloading) → 0.5mm → (unloading) → (τ =0) → (reloading) → 1.0mm → (unloading) → (τ =0)

Material constants:

f'_y: contact yielding strength = 45 N/mm^2

ω'_{lim}: Elastic limit of compressive contact displacement = 0.04mm

G_{max}: Maximum size of aggregate = 25mm

Contact density function: $\Omega(\theta) = \frac{1}{2}\cos\theta$

Effective ratio of contact: $K_\omega = 1 - \exp\left(1 - \frac{0.5G_{max}}{\omega}\right)$